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Oddness from rigidness

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Abstract: We revisit the problem of constructing type IIA orientifolds on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ which admit (non)-factorisable lattices. More concretely, we consider a $\mathbb{Z}_2 \times \mathbb{Z}_2'$ orientifold with discrete torsion, where D6-branes wrap *rigid* 3-cycles. We derive the model building rules and consistency conditions in the case where the compactification lattice is non-factorisable. We show that in this class of configurations, (semi) realistic models with an *odd* number of families can be easily constructed, in contrast to compactifications where the D6-branes wrap non-rigid cycles. We also show that an odd number of families can be obtained in the factorisable case, without the need of tilted tori. We illustrate the discussion by presenting three family Pati-Salam models with no chiral exotics in both factorisable and non-factorisable toroidal compactifications.

Keywords: Intersecting branes models, D-branes.

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1. Introduction

Type II string theory orientifold compactifications can lead to effective four dimensional theories with gauge symmetries, chiral spectrum of fermions and $\mathcal{N} = 1$ supersymmetry. Hence, they constitute a candidate string theory incorporating real particle physics. In particular, type IIA toroidal orientifolds with intersecting D-branes at angles have become extremely popular in the last years [1], due in part to their relative simplicity and thus calculability. Recent developments on these models aim to provide more realistic scenarios, as well as a better understanding of these constructions.[1]

A recent development was achieved in [6] (see also [7, 8]) where the authors considered a type IIA string theory compactified on a factorisable $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ orientifold with discrete *torsion* [9]. This type of construction admits collapsed or *rigid 3-cycles*, where intersecting D6-branes can wrap. Thus such D-branes cannot leave orbifold fixed points. This fact permitted the authors of [6] to build chiral intersecting D6-brane models with (almost) absent open string moduli. In other words, massless adjoint fields associated to the D6-brane positions can be removed from the spectrum, and asymptotic freedom is easier to achieve. However, the models studied in [6], consist of four families, which makes them phenomenologically unattractive.

An interesting generalisation to the standard factorisable IIA orientifolds usually considered in the literature [1] was performed in [10 – 12], where more general compactification

[1]Meanwhile heterotic orbifold constructions have been improved towards realistic particle physics [2 – 5].

lattices were allowed. In particular in [11] non-factorisable $T^6/\mathbb{Z}_2\times\mathbb{Z}_2$ orientifolds (without discrete torsion) were studied. In that paper, D6-brane configurations giving rise to chiral matter on the 4D spacetime were investigated. It was found that intersecting D6-brane models with non-factorisable compactification lattices, give always rise to even number of families. This observation resulted in unrealistic particle physics models, thus disfavoured in comparison with their factorisable cousins.

It is thus natural to ask whether the unsatisfactory phenomenological result found in [11], can be overcome in compactifications which admit non-factorisable lattices in addition to rigid cycles where D6-branes can wrap. This is the main question we investigate in the present paper.

We find that once rigid cycles are present, it is possible to obtain an odd number of families, as opposed to non-factorisable orientifold models without discrete torsion. Model building rules in this compactifications depend on the non-factorisable lattice, just as in the case studied in [11] for the $T^6/\mathbb{Z}_2\times\mathbb{Z}_2$ orientifold (without torison). Encouraged by these observations, we illustrate the model building rules explicitly by constructing a *three family* Pati-Salam model.[2] The model preserves $\mathcal{N}=1$ supersymmetry and contains the chiral spectrum of a three family Pati-Salam model. Mass terms for all additional fields can be written down without breaking the Pati-Salam gauge group, i.e. there are no chiral exotics.

We go beyond our original motivation and reconsider factorisable lattices of $T^6/\mathbb{Z}_2\times\mathbb{Z}_2'$ (with torsion). Following the same strategy as in the non-factorisable case, we find that factorisable lattices too admit an odd number of families. Furthermore, this has the bonus that no tilted tori are required, as it is the case with non-rigid factorisable models [17, 18]. Thus we succeed in providing examples of three-family models in factorisable and non-factorisable lattices with rigid branes, on toroidal orientifold compactifications with discrete torsion.

The paper is organised as follows. In the next section we discuss in a general setup the properties of orientifold constructions with rigid cycles valid for factorisable and non-factorisable lattices. We present tadpole constraints, spectrum and supersymmetry conditions. In section 3 we illustrate the details of the construction in a fully worked out example. We first look at a non-factorisable supersymmetric $\mathcal{N}=1$ three-family model using *rigid* visible sector branes as well as hidden *semi-rigid* and *non-rigid* branes, as will be explained in the text. We discuss tadpoles, spectrum and supersymmetry conditions. We then present the factorisable version of this model, showing how odd number of families can be obtained from rigid branes without the need of introducing tilted tori. We close in section 4 with our conclusions.

Throughout the paper, we make extensive use of the results of [6] and [11], which we advise the reader to consult for more details.

[2]We focus on the Pati-Salam instead of the Standard Model gauge group in order to automatically satisfy K-theory constrains [6]. This implies that we consider always an even number of D6-branes per stack, which then implies a gauge group U($2N$).

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2. Orientifolds with rigid cycles

In this section we describe the procedure and rules to construct intersecting D6-brane models on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ orbifolds with discrete torsion [9, 6].

Consider type IIA theory compactified on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ where the $\mathbb{Z}_2$ generators act as

$$\theta : z^{1,2} \to -z^{1,2}\,, \qquad \theta' : z^{2,3} \to -z^{2,3} \tag{2.1}$$

on the three complex coordinates of the compact space.

Extending the discussion of [6], we allow the T^6 lattice to be either factorisable or non-factorisable, i.e. the factorisation $T^6 = (T^2)^3$ is not respected by the orbifold action. Moreover, we choose our compactification such that fundamental lattice vectors can be expressed as integer linear combinations of fundamental vectors in the factorisable lattice. The factorisable lattice is a product of three T^2 lattices where each T^2 is obtained by compactification of the complex planes spanned by the coordinates appearing in (2.1). The fundamental cycles on these T^2 are denoted by $\left[a^i\right]$ and $\left[b^i\right]$, $i = 1, 2, 3$,

$$\begin{aligned} \left[a^1\right] &= (1,0,0,0,0,0)\,, & \left[b^1\right] &= (0,1,0,0,0,0)\,, \\ \left[a^2\right] &= (0,0,1,0,0,0)\,, & \left[b^2\right] &= (0,0,0,1,0,0)\,, \\ \left[a^3\right] &= (0,0,0,0,1,0)\,, & \left[b^3\right] &= (0,0,0,0,0,1)\,, \end{aligned} \tag{2.2}$$

in real coordinates, x^a, $a = 1, \ldots, 6$, which are related to the complex coordinates in (2.1) as $z^I = x^{2I-1} + \mathrm{i}\, x^{2I}$, $I = 1, 2, 3$.

It is convenient to give wrapping numbers always with respect to the factorisable basis as we do in the rest of the paper. This implies that on non-factorisable lattices not all integer wrapping numbers are allowed (see [11]).

2.1 Rigid cycles

Let us consider first the covering space T^6. We introduce D6-branes at angles, which are specified by wrapping numbers (n^i, m^i) along $[a^i]$ and $[b^i]$. Thus an orbifold invariant D6-brane labelled a wraps the three-cycle:

$$\Pi_a^{T^6} = \bigotimes_{i=1}^{3} \left(n_a^i\, [a^i] + m_a^i\, [b^i]\right). \tag{2.3}$$

As explained in [6], these cycles of T^6 are inherited by the orbifold quotient. Moreover under the action of $\mathbb{Z}_2 \times \mathbb{Z}_2'$, a three-cycle on T^6 has three images, all of them with the same wrapping numbers as the initial three-cycle. Therefore, a three-cycle can be identified with $[\Pi_a^B] = 4\,[\Pi_a^{T^6}]$. Computing the intersection number of two such cycles gives

$$[\Pi_a^B] \cdot [\Pi_b^B] = 4\,[\Pi_a^{T^6}] \cdot [\Pi_b^{T^6}] \tag{2.4}$$

where $[\Pi_a^{T^6}] \cdot [\Pi_b^{T^6}]$ has to be worked out for each non-factorisable lattice separately as was shown in [11] (see also section 3).

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Besides these untwisted cycles there are also independent collapsed three-cycles for each of the three twisted sectors, θ, θ' and $\theta\theta'$. In order to determine these, we need to know the fixed points associated to the compactification lattice. For non-factorisable tori, these have to be found in each lattice independently. We perform this counting explicitly in the next section. Here we give general expressions for a given lattice.

Consider first the θ twisted sector. We denote the location of the fixed torus on the first two complex planes by $[E^{\theta}_{I_a}]$, where I_a labels the fixed point through which a stack of branes D_a passes in this sector. For the $\mathbb{Z}_2 \times \mathbb{Z}_2'$ orbifold these fixed points correspond to collapsed two-cycles in the blown up Calabi-Yau space. These two-cycles are combined with a one-cycle in the third plane $n^3[\tilde{a}^3] + m^3[\tilde{b}^3]$ in order to form a three-cycle in the θ-twisted sector. Here, $[\tilde{a}^3]$ and $[\tilde{b}^3]$ generate the θ-fixed torus. For the factorisable lattice, they coincide with $\left[a^3\right]$ and $\left[b^3\right]$. Let us denote a basis of such twisted three-cycles as

$$[\alpha^{\theta}_{I,n}] = 2\,[E^{\theta}_I] \otimes [\tilde{a}^3], \qquad [\alpha^{\theta}_{I,m}] = 2\,[E^{\theta}_I] \otimes [\tilde{b}^3]\,. \tag{2.5}$$

The extra factor of two is due to the action of θ' on the twisted three-cycles in the third complex plane. Analogously, the basic twisted three-cycles in the θ' and $\theta\theta'$ twisted sectors are defined as

$$\begin{aligned} [\alpha^{\theta'}_{I,n}] &= 2\,[E^{\theta'}_I] \otimes [\tilde{a}^1], & [\alpha^{\theta'}_{I,m}] &= 2\,[E^{\theta'}_I] \otimes [\tilde{b}^1], \\ [\alpha^{\theta\theta'}_{I,n}] &= 2\,[E^{\theta\theta'}_I] \otimes [\tilde{a}^2], & [\alpha^{\theta\theta'}_{I,m}] &= 2\,[E^{\theta\theta'}_I] \otimes [\tilde{b}^2]. \end{aligned} \tag{2.6}$$

The intersection number between a pair of such cycles is easy to compute knowing that $[E^g_I] \cdot [E^h_J] = -2\delta_{IJ}\,\delta^{gh}$. Thus the full twisted three-cycles are given by

$$[\Pi^g_{I,a}] = n_a^{i_g}\,[\alpha^g_{I,n}] + m_a^{i_g}\,[\alpha^g_{I,m}]. \tag{2.7}$$

Given two three-cycles

$$[\Pi^g_{I,a}] = n_a^{i_g}\,[\alpha^g_{I,n}] + m_a^{i_g}\,[\alpha^g_{I,m}]$$

and

$$[\Pi^h_{J,b}] = n_b^{i_h}\,[\alpha^h_{J,n}] + m_b^{i_h}\,[\alpha^h_{J,m}]\,,$$

with $g, h = \theta, \theta', \theta\theta'$, the intersection between them is

$$[\Pi^g_{I,a}] \cdot [\Pi^h_{J,b}] \;=\; 4\,\delta_{IJ}\delta^{gh}\,(n_a^{i_g}\,m_b^{i_g} - m_a^{i_g}\,n_b^{i_g}) \;=\; 4\,\delta_{IJ}\delta^{gh}\,(n_a^{i_g}\,m_b^{i_g} - m_a^{i_g}\,n_b^{i_g}), \tag{2.8}$$

where we have again identified intersection points under the orbifold action and we have used that $[\tilde{a}^i] \cdot [\tilde{b}^j] = -\delta_{ij}$. In this notation, for the twisted sectors $g = \theta, \theta', \theta\theta'$ one has $i_g = 3, 1, 2$, respectively.

Now that we know how to describe the non-factorisable untwisted and twisted sector three-cycles, we construct rigid D6-branes in this setup. That is, we consider fractional D6-branes which are wrapping special Lagrangian 3-cycles, and are charged under all three different twisted sectors of the orbifold. The location of a fractional D6-brane has to be invariant under the orbifold action and thus it must run through four fixed points for each twisted sector. Denoting this set of fixed points as S^a_g, the fractional D-brane wraps the cycle

$$\Pi^F_a \;=\; \frac{1}{4}\,\Pi^B_a + \frac{1}{4}\sum_{I\in S^a_\theta} \epsilon^{\theta}_{a,I}\,\Pi^{\theta}_{I,a} + \frac{1}{4}\sum_{J\in S^a_{\theta'}} \epsilon^{\theta'}_{a,J}\,\Pi^{\theta'}_{J,a} + \frac{1}{4}\sum_{K\in S^a_{\theta\theta'}} \epsilon^{\theta\theta'}_{a,K}\,\Pi^{\theta\theta'}_{K,a} \tag{2.9}$$

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where the 1/4 factor indicates that one needs four fractional branes in order to get a bulk brane. Also $\epsilon^{\theta}_{a,I}$, $\epsilon^{\theta'}_{a,J}$, $\epsilon^{\theta\theta'}_{a,K} = \pm 1$ define the charge of the fractional brane a with respect to the massless fields living at the various fixed points. In the next section we consider only $\epsilon^{g}_{J} = 1$, as this is enough to illustrate our main point. However, more complicated situations can be arranged. A longer discussion can be found in [6] for the factorisable case.

2.2 Tadpoles and K-theory

We now mod out this theory by the orientifold action $\Omega\mathcal{R}$, where Ω is the world sheet parity and $\mathcal{R}$ acts by

$$\mathcal{R} : z^I \to \overline{z}^I \, .$$

This action introduces four types of O6-planes associated to the actions $\Omega\mathcal{R}$ $\Omega\mathcal{R}\theta$, $\Omega\mathcal{R}\theta'$, $\Omega\mathcal{R}\theta\theta'$. The corresponding O-plane can be either a $\mathrm{O6}^{(-,-)}$ with negative RR charge and tension or an exotic $\mathrm{O6}^{(+,+)}$ with positive RR charge and tension. Consistency with discrete torsion implies that we need to introduce an odd number of exotic O6-planes [6]. In the rest of the paper, we take a single exotic plane associated to $\mathrm{O}_{\Omega\mathcal{R}}$.

Taking this into account, we can define the homology classes of the cycles wrapped by the O6-planes as follows

$$\Pi_{O6} = \Pi_{\Omega\mathcal{R}} + \Pi_{\Omega\mathcal{R}\theta} + \Pi_{\Omega\mathcal{R}\theta'} + \Pi_{\Omega\mathcal{R}\theta\theta'} \tag{2.10}$$

where

$$\begin{aligned} \Pi_{\Omega\mathcal{R}} &\sim -2[a^1]\times[a^2]\times[a^3], & \Pi_{\Omega\mathcal{R}\theta} &\sim -2[b^1]\times[b^2]\times[a^3], \\ \Pi_{\Omega\mathcal{R}\theta'} &\sim -2[a^1]\times[b^2]\times[b^3], & \Pi_{\Omega\mathcal{R}\theta\theta'} &\sim -2[b^1]\times[a^2]\times[b^3]. \end{aligned} \tag{2.11}$$

For factorisable lattices, the $\sim$ signs in (2.11) are equality signs. For non-factorisable lattices, additional factors of two appear, if they are needed to obtain closed cycles [11].

In the rest of the paper, we consider only the **AAA** orientifold[3] for factorisable compactifications and the (related) **CCC** [11] setup for non-factorisable ones. With our conventions for the wrapping numbers (2.3), the tadpole condition

$$\sum_a N_a(\Pi^F_a + \Pi^F_{a'}) = 4\Pi_{O6} \tag{2.12}$$

can be expressed as untwisted

$$\begin{aligned} \textstyle\sum_a N_a n^1_a n^2_a n^3_a &= -16, \\ \textstyle\sum_a N_a m^1_a m^2_a n^3_a &= -16, \\ \textstyle\sum_a N_a m^1_a n^2_a m^3_a &= -16, \\ \textstyle\sum_a N_a n^1_a m^2_a m^3_a &= -16, \end{aligned} \tag{2.13}$$

[3] We are using the notation introduced in [13].

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Representation	Multiplicity
$\begin{array}{c}\Box\\[-3pt]\Box\end{array}_a$	$\frac{1}{2}\left(\Pi'_a\cdot\Pi_a+\Pi_{\mathrm{O6}}\cdot\Pi_a\right)$
$\Box\!\Box_a$	$\frac{1}{2}\left(\Pi'_a\cdot\Pi_a-\Pi_{\mathrm{O6}}\cdot\Pi_a\right)$
$(\overline{\Box}_a,\Box_b)$	$\Pi_a\cdot\Pi_b$
$(\Box_a,\Box_b)$	$\Pi'_a\cdot\Pi_b$

Table 1: Chiral spectrum for intersecting D6-branes [6].

plus twisted

$$\begin{aligned}\sum_a N_a n_a^1 \epsilon_{a,I}^{\theta'} &= 0,\\ \sum_a N_a n_a^2 \epsilon_{a,J}^{\theta\theta'} &= 0,\\ \sum_a N_a n_a^3 \epsilon_{a,K}^{\theta} &= 0,\end{aligned} \tag{2.14}$$

tadpole constraints. The minus sign on the r.h.s. of the first equation in (2.13) reflects the appearance of an exotic O-plane in the case with discrete torsion [6]. As explained in [11] the number of O-planes is reduced in non-factorisable lattices. However, for some wrapping numbers one unit corresponds to a half-cycle as they refer to cycles on the factorisable lattice. These two effects cancel resulting in the universal expressions (2.13), (2.14). The lattice dependence arises due to the fixed point structure.

The tadpole conditions ensure the cancellation of non-Abelian anomalies. On top of that, one has to impose K-theory constraints [14]. As discussed for example in the appendix of [6] these imply that a probe SU(2) stack of branes must host an even number of fundamentals of SU(2). Following their lead, we impose the sufficient condition that all our stacks contain an even number of branes.

2.3 Spectrum

The resulting spectrum can now be calculated, as has been done in [6]. We reproduce it here for completeness. Firstly, D6-branes wrapping three-cycles not invariant under $\Omega\mathcal{R}$ give rise to the gauge group $\mathrm{U}(N_a)$. If two such branes intersect at an angle open strings stretched between them will have massless excitations. These give rise to chiral multiplets transforming under the product of the two gauge groups on the branes. The resulting massless spectrum is given in table 1, where also the situation that brane D_a intersects with its orientifold image $D_{a'}$ is included. In the latter case there is only one gauge group factor due to the orientifold identification. Further, branes that are invariant under the orientifold action $\Omega\mathcal{R}\Pi_a^F=\Pi_a^F$ do not yield a unitary group but rather a simplectic group $\mathrm{USp}(2N_a)$. In the $\mathbb{Z}_2\times\mathbb{Z}_2'$ orbifold, fractional branes invariant under $\Omega\mathcal{R}$ are those placed on top of an exotic $\mathrm{O6}^{(+,+)}$ plane. In our choice, they sit on top of the $\mathrm{O}_{\Omega\mathcal{R}}$ plane (see [6] for further details). Finally we recall that no adjoint fields from an aa sector arise for rigid branes.

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2.4 Supersymmetry

Although intersecting brane models which break supersymmetry explicitly are not necessarily inconsistent, they usually suffer from instabilities. In order to avoid that to happen, we focus on models with residual $\mathcal{N} = 1$ supersymmetry. This amounts to the condition that the angles θ_a^I $(I = 1, 2, 3)$ every brane D_a forms with the horizontal coordinate axes in each complex plane have to add up to zero [15],

$$\theta_1 + \theta_2 + \theta_3 = 0 \mod 2\pi. \tag{2.15}$$

Often metric moduli can be adjusted such that (2.15) is satisfied. For later use, we specify the metric of the compact space G_{ab} $(a, b = 1, \ldots, 6)$ to be diagonal[4] in the coordinate basis of the x^a (with $z^I = x^{2I-1} + \mathrm{i}\, x^{2I}$ being the complex coordinates in (2.1)) and define

$$U^I = \sqrt{\frac{G_{2I,2I}}{G_{2I-1,2I-1}}}, \quad I = 1, 2, 3. \tag{2.16}$$

For the factorisable lattice the U^I are the complex structure moduli of the T^2 factors.

3. Explicit models

In this section we consider a concrete model which serves to illustrate the model building rules, as well as how the number of families restriction can be implemented once rigid branes are introduced. We do this in detail in a simple non-factorisable lattice which serves to demonstrate our main result. We then construct the factorisable version of the same model, in order to show how three family models arise in that case too. Besides the family requirement, we also need to impose twisted and untwisted tadpole conditions as well as supersymmetry to the models. These constraints impose strong conditions on the brane wrapping numbers.

3.1 Non-factorisable lattice

As a minimal non-factorisable example, consider a lattice $\{e_i\}$ where the third and fifth lattice vectors are given by

$$e_3 = (0, 0, 1, 0, -1, 0) \quad , \quad e_5 = (0, 0, 1, 0, 1, 0) \tag{3.1}$$

and keep the rest in a factorisable form (**AAA** lattice). Employing the Lefschetz fixed point theorem one finds that there are 8 θ-fixed tori, 16 θ'-fixed tori and 8 $\theta\theta'$ fixed tori. The 8 θ-fixed tori are (underlined entries can be permuted)

$$\begin{gathered}(0, 0, 0, 0, x, y), \qquad \left(\underline{\frac{1}{2}, 0}, 0, \underline{0}\,, x, y\right), \\ \left(\underline{\frac{1}{2}, \frac{1}{2}}, 0, \underline{0}\,, x, y\right), \ \left(\frac{1}{2}, \frac{1}{2}, 0, \frac{1}{2}, x, y\right).\end{gathered} \tag{3.2}$$

[4] Off-diagonal components are projected out.

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Here, x and y are compactified on a two dimensional lattice generated by $(2,0)$ and $(0,1)$. The 16 θ'-fixed tori are

$$\begin{aligned}
&(x,y,0,0,0,0), && \left(x,y,0,\frac{1}{2},0,0\right), && \left(x,y,0,\frac{1}{2},0,\frac{1}{2}\right), \\
&\left(x,y,\frac{1}{2},0,-\frac{1}{2},0\right), && \left(x,y,\frac{1}{2},\frac{1}{2},-\frac{1}{2},0\right), && \left(x,y,\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right), \\
&\left(x,y,\frac{1}{2},0,\frac{1}{2},0\right), && \left(x,y,\frac{1}{2},\frac{1}{2},\frac{1}{2},0\right), && \left(x,y,\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\right), \\
&(x,y,1,0,0,0), && \left(x,y,1,\frac{1}{2},0,0\right), && \left(x,y,1,\frac{1}{2},0,\frac{1}{2}\right).
\end{aligned} \tag{3.3}$$

Now, the compactification lattice for (x,y) is generated by $(1,0)$ and $(0,1)$. Finally the 8 $\theta\theta'$-fixed tori are

$$\begin{aligned}
&(0,0,x,y,0,0), && \left(\frac{1}{2},0,x,y,0,0\right), \\
&\left(\frac{1}{2},\frac{1}{2},x,y,0,0\right), && \left(\frac{1}{2},\frac{1}{2},x,y,0,\frac{1}{2}\right),
\end{aligned} \tag{3.4}$$

where the compactification lattice for (x,y) is generated by $(2,0)$ and $(0,1)$.

Let us now compute the intersection number between two rigid D6-branes given a compactification lattice. To do this, remember first that we denote a D6-brane by its bulk wrapping numbers as (2.3) [11]:

$$D6_a = \left(m_a^1\left[a^1\right] + n_a^1\left[b^1\right]\right) \times \left(m_a^2\left[a^2\right] + n_a^2\left[b^2\right]\right) \times \left(m_a^3\left[a^3\right] + n_a^3\left[b^3\right]\right), \tag{3.5}$$

where the one-cycles are listed in (2.2), and m_a^i, n_a^i $(i = 1,2,3)$ are integers, we see that the cycle (3.5) is closed on the compactification lattice if

$$n_a^2 = \text{even} \quad \text{and} \quad n_a^3 = \text{even}, \tag{3.6}$$

otherwise the brane has to wrap the corresponding cycle of the factorisable lattice twice [11].

Now, the contribution from the bulk piece can be expressed as:

$$[\Pi_a^B]\cdot[\Pi_b^B] = 4\,[\Pi_a^{T^6}]\cdot[\Pi_b^{T^6}] = 2\prod_{i=1}^{3}(n_a^i m_b^i - m_a^i n_b^i) \tag{3.7}$$

where we have used the results in [11] to compute the intersection number $[\Pi_a^{T^6}]\cdot[\Pi_b^{T^6}]$. Adding the contribution from the twisted parts, using (2.8) and (2.9), we find that the general expression for the intersection number between fractional branes in the present lattice can be written as follows:

$$\begin{aligned}
I_{ab} = {} & \frac{1}{8}\prod_{i}^{3}(n_a^i m_b^i - m_a^i n_b^i) + \frac{\delta_{ab}^{\theta}}{4}\left(\frac{n_a^3}{2}m_b^3 - m_a^3\frac{n_b^3}{2}\right) + \frac{\delta_{ab}^{\theta'}}{4}\left(n_a^1 m_b^1 - m_a^1 n_b^1\right) + \\
& + \frac{\delta_{ab}^{\theta\theta'}}{4}\left(\frac{n_a^2}{2}m_b^2 - m_a^2\frac{n_b^2}{2}\right),
\end{aligned} \tag{3.8}$$

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where δ^g_{ab} denotes the number of common g-fixed points between brane stacks a and b.

Computing the net number of families[5] $I_{ab} - I_{a'b}$,

$$\begin{aligned} I_{ab} - I_{a'b} = -\frac{1}{4}\Big[& m^3_a n^3_b m^1_b m^2_b n^1_a n^2_a + m^1_a n^1_b m^2_b m^3_b n^2_a n^3_a + m^1_a n^1_b m^2_a m^3_a n^2_b n^3_b + \\ & + m^2_a n^2_b m^1_b m^3_b n^1_a n^3_a + m^3_a n^3_b \, \delta^{\theta}_{ab} + m^1_a n^1_b \, \delta^{\theta'}_{ab} + m^2_a n^2_b \, \delta^{\theta\theta'}_{ab} \Big] , \end{aligned} \tag{3.9}$$

it can be seen that odd numbers can be easily obtained. Indeed, one can check that if two branes have less than four fixed points in common in some sectors, that is $\delta^g_{ab} \neq (4,4,4)$, as well as requiring suitable m^i's to be odd, it is possible to have an odd number of families. We show later that a similar condition applies to the factorisable case.

Finally in order to fully compute the spectrum, we need the intersection between the O6-planes and the fractional branes. In the present compactification lattice, the cycles wrapped by the O6-planes (2.10), (2.11) can be written as [11]

$$\begin{aligned} \Pi_{O6} = 2\Big[& (-1,0)\times(1,0)\times(2,0) + (0,1)\times(0,-1)\times(2,0) \\ & + (1,0)\times(0,1)\times(0,-1) + (0,1)\times(2,0)\times(0,-1) \Big] , \end{aligned} \tag{3.10}$$

where the sign in the first contribution comes from the exotic O6-plane. Then the intersection between the O6-planes with the branes can be computed using the results in [11], and boils down to the following expression

$$\Pi_{O6}\cdot\Pi^F_a = \Pi_{O6}\cdot\Pi^{T^6}_a = \sum_{Oj}\prod_i (n^i_{Oj} m^i_a - m^i_{Oj} n^i_a) , \tag{3.11}$$

where n^i_{Oj} correspond to 'wrapping numbers' for the O6-planes (3.10) and the sum is over the four types of O6-planes.

3.2 Three family Pati-Salam model

We are now ready to construct a three family Pati-Salam model using rigid as well as hidden semi-rigid and non-rigid branes. As discussed in [6], sometimes rigid branes can combine with other rigid branes to form a bulk brane which can move off the fixed points. Moduli in the adjoint of a gauge group reappear when this happens. Thus, such a set of branes forms a non-rigid brane. Branes which can be combined into a bulk brane have the same wrapping numbers and cancelling twisted charges (see eq. (2.14)). We call branes with the same wrapping numbers and cancelling twisted charges in one twisted sector, semi-rigid. These can combine and form a brane which can move away from the fixed points only in some directions.

A reversed view of this definition starts with a bulk brane. If its location is invariant under the orbifold, it can split into its four fractional pieces obtained by separating θ, θ', $\theta\theta'$ images and adding contributions from collapsed cycles such that each piece forms a closed cycle in the blown up orbifold (see eq. (2.9) and [16]). Keeping all such fractional

[5] We consider the case that the colour group is a subgroup of the gauge symmetry on stack a.

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pieces results in a set which we call non-rigid, while keeping only the images of one $\mathbb{Z}_2$ factor, yields a semi-rigid set.

Let us start by describing the model building strategy. We have seen that in order to get an odd number of families, it is necessary to have some of the fixed points different from their maximum value, that is $\delta^g_{ab} \neq (4,4,4)$ (four being the maximum in each entry). Therefore, in order to cancel twisted tadpoles at all fixed points, it will be necessary to introduce additional branes, compared to the case when all fixed points are shared between branes[6] (that is, when $\delta^g_{ab} = (4,4,4)$). Care will be taken such that these extra branes do not introduce exotic chiral matter. A priori one will attach them to the hidden sector. However, in order to obtain massless GUT Higgs pairs in the spectrum, it will be necessary to recombine one stack of the additional branes with the stack carrying initially the $\mathrm{SU}(2)_R$ gauge symmetry factor of the Pati-Salam group (see below). Finally, we will be interested in models which preserve $\mathcal{N} = 1$ supersymmetry. This will constrain further the wrapping numbers of the brane stacks and fix some closed string moduli.

More explicitly, consider first a set of three *rigid branes* $\{a_1, a_2, a_3\}$, the (a priori) visible sector, which share some, but not all, fixed points in some sectors. In general this leads to some uncancelled twisted tadpoles among themselves. Therefore it is necessary to introduce an (a priori) hidden set of branes, such that the twisted tadpoles are cancelled. In order to minimise this, the two branes $\{a_2, a_3\}$ that will give rise to the gauge groups $\mathrm{SU}(2)_{L,R}$ in the Pati-Salam model, are taken such that they share exactly the same set of fixed points, i.e. $\delta^g_{a_2 a_3} = (4,4,4)$. Hence, each one will contribute to the same kind of twisted tadpoles, and we choose them such that these tadpoles are cancelled between them. Thus we are left with uncancelled twisted tadpoles only from the stack $\{a_1\}$. In order to cancel these, we introduce a set of stacks $\{b_i\}$, such that *all* twisted tadpoles from sets $\{a, b\}$ are cancelled. The set of branes in all stacks $\{b_i\}$ have the same wrapping numbers and twisted charges with respect to one of the $\mathbb{Z}_2$ factors, so that twisted tadpoles are cancelled among them. Thus they form a stack of *semi-rigid* branes. Cancellation of untwisted tadpoles can at last be achieved by introducing suitable sets of hidden sector branes, without introducing new contributions to the twisted tadpoles. In the model we construct below, two more of these sets $\{c, d\}$ will be needed. Stacks within each of the sets $\{c\}$ can combine into bulk branes and hence they form *non-rigid* stacks. Indeed, it is the requirement of unbroken residual supersymmetry which restricts us to consider *all* hidden sector branes to be semi-rigid or non-rigid.

Taking into account all the requirements listed above, we end up with the semi realistic Pati-Salam-like model specified in table 2. We perform in detail its analysis in what follows.

Let us start by identifying the fixed points through which the visible and hidden sector branes pass. These are explicitly listed in table 3. Next, we construct the basis of

[6] An easy way to cancel twisted tadpoles is to consider only branes which share all four fixed points, that is $\delta^g_{ab} = (4,4,4)$. In such case, it is enough to fix appropriately the values of the wrapping numbers n^i such that no tadpoles are left uncancelled (see (2.14)). This trick was used in the four family models constructed in [6].

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N_α	(n^1_α, m^1_α)	(n^2_α, m^2_α)	(n^3_α, m^3_α)
$N_{a_1} = 4$	$(0, 1)$	$(0, -1)$	$(2, 0)$
$N_{a_2} = 2$	$(-1, 1)$	$(4, -3)$	$(0, -1)$
$N_{a_3} = 2$	$(1, -3)$	$(-4, 1)$	$(0, -1)$
$N_{b_1} = 2$	$(-4, -1)$	$(-4, -1)$	$(-2, 1)$
$N_{b_2} = 2$	$(4, 1)$	$(4, 1)$	$(-2, 1)$
$N_{c_1} = 14$	$(1, 0)$	$(1, 0)$	$(2, 0)$
$N_{c_2} = 14$	$(-1, 0)$	$(-1, 0)$	$(2, 0)$
$N_{c_3} = 14$	$(1, 0)$	$(-1, 0)$	$(-2, 0)$
$N_{c_4} = 14$	$(-1, 0)$	$(1, 0)$	$(-2, 0)$
$N_{d_1} = 12$	$(1, 0)$	$(0, 1)$	$(0, -1)$
$N_{d_2} = 12$	$(-1, 0)$	$(0, 1)$	$(0, 1)$

Table 2: Wrapping numbers for the three family non-factorisable Pati-Salam model.

the twisted three-cycles as defined in (2.5). For brane $\{a_1\}$, the basis is given by

$$\left[\alpha^{\theta}_{I_{a1},\,n}\right] = 2\,\left[E^{\theta}_{I_{a1}}\right] \otimes [0,0,0,0,2,0]\ , \tag{3.12}$$

$$\left[\alpha^{\theta'}_{I_{a1},\,m}\right] = 2\,[0,1,0,0,0,0] \otimes \left[E^{\theta'}_{I_{a1}}\right]\ , \tag{3.13}$$

$$\left[\alpha^{\theta\theta'}_{I_{a1},\,m}\right] = 2\,[0,0,0,1,0,0] \otimes \left[E^{\theta\theta'}_{I_{a1}}\right]\ , \tag{3.14}$$

where $[E^{g}_{I_{a1}}]$[7] correspond to the 4 fixed points associated to brane $\{a_1\}$ in each sector. These are listed in the first column of table 3. From this basis, we can construct the twisted 3-cycle which the brane wraps, using (2.7):

$$[\Pi^{\theta}_{I,\,a_1}] = 1\cdot[\alpha^{\theta}_{I_{a1},n}]\,, \qquad [\Pi^{\theta'}_{I,\,a_1}] = 1\cdot[\alpha^{\theta'}_{I_{a1},m}]\,, \qquad [\Pi^{\theta\theta'}_{I,\,a_1}] = -1\cdot[\alpha^{\theta\theta'}_{I_{a1},m}]\,. \tag{3.15}$$

Finally, the full fractional cycle (2.9), which the stack $\{a_1\}$ wraps is given by

$$\Pi^F_{a_1} \;=\; \frac{1}{4}\,\Pi^B_{a_1} + \frac{1}{4}\sum_I^4 \Pi^{\theta}_{I,\,a_1} + \frac{1}{4}\sum_I^4 \Pi^{\theta'}_{I,\,a_1} + \frac{1}{4}\sum_I^4 \Pi^{\theta\theta'}_{I,\,a_1}. \tag{3.16}$$

For stacks $\{a_i\}$ $(i = 2, 3)$, we have instead

$$[\alpha^{\theta}_{I_{a_i},\,m}] = 2\,[E^{\theta}_{I_{a_i}}] \otimes [0,0,0,0,0,1]\,, \tag{3.17}$$

[7] We are being sloppy here, using the same symbol to denote fixed points, tori or cycles. However, it should be clear from the context what we are referring to.

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$$[\alpha^{\theta'}_{I_{a_i},n}] = 2\,[1,0,0,0,0,0] \otimes [E^{\theta'}_{I_{a_i}}]\,, \qquad [\alpha^{\theta'}_{I_{a_i},m}] = 2\,[0,1,0,0,0,0] \otimes [E^{\theta'}_{I_{a_i}}]\,, \tag{3.18}$$
$$[\alpha^{\theta\theta'}_{I_{a_i},n}] = 2\,[0,0,2,0,0,0] \otimes [E^{\theta\theta'}_{I_{a_i}}]\,, \qquad [\alpha^{\theta\theta'}_{I_{a_i},m}] = 2\,[0,0,0,1,0,0] \otimes [E^{\theta\theta'}_{I_{a_i}}]\,, \tag{3.19}$$

where $[E^{g}_{I_{a_i}}]$ correspond to the four fixed points associated to brane $\{a_i\}$ in each sector (see table 3). The twisted 3-cycle which the brane $\{a_2\}$ wraps (stack $\{a_3\}$ is very similar) is then:

$$\begin{aligned}
[\Pi^{\theta}_{I,\,a_2}] &= -1 \cdot [\alpha^{\theta}_{I_{a2},\,n}]\,, \\
\left[\Pi^{\theta'}_{I,\,a_2}\right] &= -1 \cdot [\alpha^{\theta'}_{I_{a1},\,m}] + 1 \cdot [\alpha^{\theta'}_{I_{a2},\,m}]\,, \\
\left[\Pi^{\theta\theta'}_{I,\,a_2}\right] &= 4 \cdot [\alpha^{\theta\theta'}_{I_{a2},\,n}] - 1 \cdot [\alpha^{\theta\theta'}_{I_{a2},\,m}]\,.
\end{aligned} \tag{3.20}$$

Thus, the full fractional cycle (2.9), which the stack $\{a_2\}$ wraps is given by (again, stack $\{a_3\}$ is very similar)

$$\Pi^{F}_{a_2} = \frac{1}{4}\,\Pi^{B}_{a_2} + \frac{1}{4}\sum_{I}^{4} \Pi^{\theta}_{I,\,a_2} + \frac{1}{4}\sum_{I}^{4} \Pi^{\theta'}_{I,\,a_2} + \frac{1}{4}\sum_{I}^{4} \Pi^{\theta\theta'}_{I,\,a_2}. \tag{3.21}$$

For all other branes, one can find the fractional cycles in a similar fashion.

Before proceeding to calculate the spectrum, we need to clarify some subtleties regarding the fixed points denoted with a $\star$ in table 3. Consider for example the point $(0,0,1,0)^\star$ in the $\theta\theta'$ sector of brane $\{a_1\}$ (see table 3). Suppose it denoted the locus of a fixed torus, as in section 3.1, then it would be equivalent to zero. However, here we are looking at the one-cycle (or collapsed three-cycle) wrapped by the D-brane and it matters in which direction the one-cycle extends. Consider the full cycle $(0,0;0,-x;1,0)$ for brane $\{a_1\}$, this is equivalent to $(0,0;1,-x;0,0)$ and it is therefore shifted in the third direction. One has to take this into account when counting the number of common fixed points between a pair of branes. In computing the intersection number between two branes, the shifted (second) version has to be used. If the brane extended along the third direction instead, the fixed cycle would indeed be equivalent to the one located at the origin and contribute only once to the counting of common fixed points.

We are now ready to calculate the chiral spectrum arising from the Pati-Salam stacks of branes $\{a,\ b\}$ and the auxiliary branes $\{c,\ d\}$. For reasons mentioned already and to be discussed shortly, we assign the visible sector to the set $\{a_1, a_2, a_3, b_1\}$. The spectrum arising from open strings stretched between different branes within this set is displayed in table 4, where we removed anomalous $\mathrm{U}(1)$ factors from the gauge groups.

Notice further that the stack of branes $\{b\}$ has been arranged such that, not only the $\{a_1\}$ twisted tadpoles are cancelled, but also such that the net intersection between branes $\{b\}$ and $\{a_1\}$ vanishes. Second, chiral matter arising from possible intersections between brane $\{a_1\}$ and branes $\{c,\ d\}$ is eliminated by shifting the latter branes away from the origin, such that the twisted contribution, as well as the bulk parts of the intersection numbers vanish (this possibility was also used in the models of [6]). Thus, no extra chiral matter charged under the Pati-Salam $\mathrm{U}(4)$ arises.

Now let us look at some of the phenomenological implications of the model. As far as the Standard Model matter and the electroweak Higgs is concerned, it would have been

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θ sector	a_1	$a_{2,3}$	b_i	c_i	d_i
E_1^θ	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)
E_2^θ	(0,1/2,0,0)	(1/2,1/2,0,0)	(0,1/2,0,0)	$(0,0,1,0)^\star$	(1/2,0,0,0)
E_3^θ	(0,0,0,1/2)	(0,0,0,1/2)	(0,0,0,1/2)	(1/2,0,0,0)	(0,0,0,1/2)
E_4^θ	(0,1/2,0,1/2)	(1/2,1/2,0,1/2)	(0,1/2,0,1/2)	$(1/2,0,1,0)^\star$	(1/2,0,0,1/2)
θ' sector					
$E_1^{\theta'}$	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)
$E_2^{\theta'}$	(0,1/2,0,0)	(0,1/2,0,0)	(0,1/2,0,0)	(1,0,0,0)	(0,1/2,0,0)
$E_3^{\theta'}$	(1,0,0,0)	(0,0,0,1/2)	(1,0,0,1/2)	(0,0,1,0)	(0,0,0,1/2)
$E_4^{\theta'}$	(1,1/2,0,0)	(0,1/2,0,1/2)	(1,1/2,0,1/2)	(1/2,0,1/2,0)	(0,1/2,0,1/2)
$\theta\theta'$ sector					
$E_1^{\theta\theta'}$	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)
$E_2^{\theta\theta'}$	$(0,0,1,0)^\star$	(1/2,1/2,0,0)	(0,1/2,0,0)	$(0,0,1,0)^\star$	(1/2,0,0,0)
$E_3^{\theta\theta'}$	(0,1/2,0,0)	(0,0,0,1/2)	(0,0,0,1/2)	(1/2,0,0,0)	(0,0,0,1/2)
$E_4^{\theta\theta'}$	$(0,1/2,1,0)^\star$	(1/2,1/2,0,1/2)	(0,1/2,0,1/2)	$(1/2,0,1,0)^\star$	(1/2,0,0,1/2)

Table 3: Fixed points for the non-factorisable branes in the Pati-Salam model of table 2.

Sector	$\mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$	$\mathrm{SU}(2) \times \mathrm{USp}(28)^4 \times \mathrm{SU}(12)^2$
$(a_1\, a_2)$	$3 \times (\overline{4}, 2, 1, 1)$	$(1; 1, 1, 1, 1; 1, 1)$
$(a_1\, a_3)$	$3 \times (4, 1, 2, 1)$	''
$(a_2\, a_3)$	$14 \times (1, 2, 2, 1)$	''
$(a'_2\, a_2)$	$14 \times (1, 1, 1, 1)$	''
$(a'_3\, a_3)$	$12 \times (1, 1, 1, 1) + 2 \times (1, 1, 3, 1)$	''
$(a_1\, b_1)$	$3 \times (4, 1, 1, 2)$	''
$(a'_1\, b_1)$	$3 \times (\overline{4}, 1, 1, 2)$	''
$(a_2\, b_1)$	$23 \times (1, 2, 1, 2)$	''
$(a_3\, b_1)$	$15 \times (1, 1, 2, 2)$	''
$(b_1\, b'_1)$	$6 \times (1, 1, 1, 3) + 16 \times (1, 1, 1, 1)$	''

Table 4: Model of table 2: Massless spectrum from open strings stretching between different branes within the 'visible sector set' $\{a, b_1\}$.

enough to consider the branes of set $\{a\}$ as the observable sector, and to identify the $\mathrm{SU}(2)_1$ with $\mathrm{SU}(2)_R$ of the Pati-Salam model. However, the GUT Higgs pair allowing

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to break the Pati-Salam group spontaneously to the Standard Model group, would be missing. Attaching the stack $\{b_1\}$ to the visible sector yields a way to get GUT Higgs pairs as well. Provided the potential is such that we can turn on vev's for bi-fundamentals of $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$ the product of the two SU(2)'s can be broken to its diagonal subgroup. Identifying that diagonal subgroup with $\mathrm{SU}(2)_R$ we obtain a Pati-Salam model with three generations of quarks and leptons as well as providing pairs of electroweak and GUT Higgses. In our example model, there will be a surplus of Higgs pairs of both types.

If this mechanism is realised, we arrive at an interesting conclusion. The requirements of obtaining three generations for quarks and leptons as well as the presence of GUT Higgs pairs in the massless spectrum are connected. To obtain three generations we had to leave some of the twisted tadpoles arising from branes hosting standard model matter uncancelled. The extra branes needed for twisted tadpole cancellation now also contribute the GUT Higgs pair to the spectrum. Choosing instead of the stack $\{b_1\}$ the stack $\{b_2\}$ would give a very similar way of obtaining the GUT Higgs pairs.

Hence, the final gauge group arising from the visible sector is, as shown in table 4. On the other hand, the hidden sector yields the gauge groups $\mathrm{U}(2) \times \mathrm{USp}(28)^2 \times \mathrm{U}(12)^2$. However, by taking some flat directions we can deform these semi and non-rigid branes into bulk D-branes. Then the final gauge group, upon eliminating anomalous $\mathrm{U}(1)$ factors is $\mathrm{SU}(2) \times \mathrm{USp}(28) \times \mathrm{SU}(12)$.

Finally, we look at supersymmetry. This imposes, from branes $\{a_2,\, a_3\}$ the condition

$$\arctan U^1 + \arctan \frac{3\,U^2}{4} = \frac{\pi}{2}\,; \qquad \arctan 3\,U^1 + \arctan \frac{U^2}{4} = \frac{\pi}{2}\,. \tag{3.22}$$

These two conditions provide the same relation between U^1, U^2, namely:

$$U^1 = \frac{4}{3\,U^2}\,. \tag{3.23}$$

On the other hand, supersymmetry on branes $\{b\}$ requires

$$\arctan \frac{U^1}{4} + \arctan \frac{U^2}{4} = \pi + \arctan \frac{U^3}{2}\,. \tag{3.24}$$

Plugging condition (3.23) into this expression gives

$$\arctan \frac{1}{3\,U^2} + \arctan \frac{U^2}{4} = \pi + \arctan \frac{U^3}{2}\,, \tag{3.25}$$

which has a non trivial solution

$$U^3 = \frac{8 + 6(U^2)^2}{11\,U^2}\,. \tag{3.26}$$

The other hidden branes $\{c,\ d\}$ do not give new constraints.

3.3 The factorisable orbifold

In this section we show that, following the same strategy as in the previous section, it is possible to get a three family left-right symmetric model from factorisable lattices with discrete torsion, *without* the need of introducing tilted tori, as in the case without torsion [17, 18].

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N_α	(n^1_α, m^1_α)	(n^2_α, m^2_α)	(n^3_α, m^3_α)
$N_{a_1} = 4$	$(0, 1)$	$(0, -1)$	$(1, 0)$
$N_{a_2} = 2$	$(-1, 1)$	$(4, -3)$	$(0, -1)$
$N_{a_3} = 2$	$(1, -3)$	$(-4, 1)$	$(0, -1)$
$N_{b_1} = 2$	$(-4, -1)$	$(-4, -1)$	$(-1, 1)$
$N_{b_2} = 2$	$(4, 1)$	$(4, 1)$	$(-1, 1)$
$N_{c_1} = 12$	$(1, 0)$	$(1, 0)$	$(1, 0)$
$N_{c_2} = 12$	$(-1, 0)$	$(-1, 0)$	$(1, 0)$
$N_{c_3} = 12$	$(1, 0)$	$(-1, 0)$	$(-1, 0)$
$N_{c_4} = 12$	$(-1, 0)$	$(1, 0)$	$(-1, 0)$
$N_{d_1} = 4$	$(0, 1)$	$(0, -1)$	$(1, 0)$
$N_{d_2} = 4$	$(0, 1)$	$(0, 1)$	$(-1, 0)$
$N_{e_1} = 12$	$(1, 0)$	$(0, 1)$	$(0, -1)$
$N_{e_2} = 12$	$(-1, 0)$	$(0, 1)$	$(0, 1)$

Table 5: Wrapping numbers for Pati-Salam model in the factorisable version of 2.

That this is the case, can be easily seen from the analogue of (3.9) in the factorisable case. This is simply:

$$I_{ab} - I_{a'b} = -\frac{1}{2}\Big[m^3_a n^3_b m^1_b m^2_b n^1_a n^2_a + m^1_a n^1_b m^2_b m^3_b n^2_a n^3_a + m^1_a n^1_b m^2_a m^3_a n^2_b n^3_b + \\ + m^2_a n^2_b m^1_b m^3_b n^1_a n^3_a + m^3_a n^3_b\, \delta^{\theta}_{ab} + m^1_a n^1_b\, \delta^{\theta'}_{ab} + m^2_a n^2_b\, \delta^{\theta\theta'}_{ab}\Big]\,. \quad (3.27)$$

From this expression it becomes clear that once some of the δ^g_{ab}'s are taken different from its maximum value, i.e. $\delta^g_{ab} \neq (4, 4, 4)$, one can get odd numbers of families (again, combined with suitable choices of the wrapping numbers). Moreover, it is also easy to see from this expression why the models considered in [6] gave always even number of families.

As an explicit example, we consider the factorisable version of the non-factorisable three family model discussed in the previous section. The wrapping numbers and brane content are listed in table 5. In this case, one can easily get the fixed point structure, fractional cycles and intersection numbers using the results of [6]. For the spectrum, we simply show the factorisable analogue of table 4 in table 6.

Notice that, compared with the same type of model in the previous section, in the factorisable case we need to introduce one extra stack of auxiliary branes $\{e\}$, in order to fully cancel untwisted tadpoles. In this respect, the non-factorisable model is more attractive.

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Sector	$\mathrm{SU}(4)\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_1\times\mathrm{SU}(2)_2$	$\mathrm{SU}(2)\times\mathrm{USp}(24)^4\times\mathrm{SU}(4)^2\times\mathrm{SU}(12)^2$
$(a_1\,a_2)$	$3\times(\overline{4},2,1,1)$	$(1;1,1,1,1;1,1;1,1)$
$(a_1\,a_3)$	$3\times(4,1,2,1)$	''
$(a_2\,a_3)$	$26\times(1,2,2,1)$	''
$(a_2'\,a_2)$	$6\times(1,3,1,1)+20\times(1,1,1,1)$	''
$(a_3'\,a_3)$	$14\times(1,1,1,1)$	''
$(a_1\,b_1)$	$3\times(4,1,1,2)$	''
$(a_1'\,b_1)$	$3\times(\overline{4},1,1,2)$	''
$(a_2\,b_1)$	$23\times(1,2,1,2)$	''
$(a_3\,b_1)$	$15\times(1,1,2,2)$	''
$(b_1\,b_1')$	$18\times(1,1,1,1)$	''

Table 6: Model of table 5: Massless spectrum from open strings stretching between different branes within the 'visible sector set' $\{a,b_1\}$.

Notice also that, as in the previous section and in [6], intersections of the auxiliary branes with the U(4) brane are cancelled off by shifting those branes away from the origin. Furthermore, the supersymmetry conditions (3.23), (3.26) are the same for this case.

4. Discussion

Motivated by the recent advances in intersecting D-brane model building, we studied $T^6/\mathbb{Z}_2\times\mathbb{Z}'_2$ orientifolds in type IIA which admit rigid cycles and (non)-factorisable lattices.

We have shown that brane pairs which do not pass through the same set of fixed points, together with suitable choices of the wrapping numbers, allow for constructions of three family non-factorisable models with semi realistic particle spectra. We demonstrated this explicitly in an $\mathcal{N}=1$, three family, Pati-Salam example. There are no chiral exotics, Pati-Salam invariant mass terms for all exotics are allowed. So, at the present stage, there are no obvious reasons against the possibility that all exotic matter decouples. In addition to the requirement of three families and no chiral exotics, tadpole cancellation and supersymmetry impose strong constraints on the wrapping numbers for the brane configurations. Hence one may expect only few models with all these characteristics to be available.

A question which needs to be addressed is to actually check whether vector-like exotics can be decoupled. For that one needs to analyse the superpotential in the effective theory as it arises from the concrete intersecting brane model. However, we emphasise that our original motivation was not to get a fully realistic model at this stage, but to show how non-factorisable lattices can give rise to three generation models.

Another interesting feature of the model we studied is that the same branes which are needed for cancelling the twisted tadpoles, $\{b\}$, also produce the GUT Higgses needed to break the Pati-Salam group down to the Standard Model group (these were not present in [6] if viewed as a four family model). Thus, these extra branes are not just needed for twisted tadpole cancellation but also for phenomenological reasons. However, as discussed in the text, the mechanism requires non-zero vev's for some scalars. Again, it would be

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desirable to turn on that vev under good knowledge of the superpotential. (Since the corresponding multiplet is massless, it is conceivable that there is indeed a flat direction along the required vev.)

As a fortunate byproduct of our study of non-factorisable lattices, we have found that the very same strategy to get odd number of families works equally well for the factorisable case without the need to introduce tilted tori as it is necessary in the case of non-rigid D6-brane models [17, 18]. We showed this in the example of a factorisable version of the Pati-Salam non-factorisable model presented. The matter and gauge group content is very similar to the non-factorisable case. However, the factorisable lattice requires the introduction of one extra set of hidden branes, $\{e\}$, in order to fully satisfy untwisted tadpole conditions. This in turn gives rise to a larger gauge group as well as further extra matter. In this respect, the non-factorisable version of the Pati-Salam model we have studied is favoured.

We expect the same trick to get odd number of families for other non-factorisable lattices to continue being valid, upon appropriate choice of the wrapping numbers. It is also plausible that other non-factorisable lattices will require less number of hidden branes in order to fully cancel tadpoles. Compared to our minimal choice, however, the rank of the gauge group will be reduced and it might become harder to embed the Standard Model gauge group.

We have just started exploration of these type of models, and thus our results are far from exhaustive. There are still several open problems that need investigation. For example, we did not touch on the issue of introducing fluxes along the lines of [6], to stabilise some of the closed string moduli. Further, we concentrated on a Pati-Salam model in order to sidestep the problem of imposing K-theory constraints, which are automatically satisfied when the number of branes per stack is even. It would be important to explore possible strategies to minimise the number of K-theory constraints such that three family MSSM like models can be investigated (see for instance [19]).

In the case of heterotic compactifications on non-factorisable lattices (for recent studies see [20 − 22]) it has been observed that the same massless spectra can be obtained from factorisable orbifolds together with a generalised notion of discrete torsion [23]. If that observation is caused by some deeper relation between generalised discrete torsion and non-factorisable compactifications it would be interesting to find a type II analogue. Such relations can yield important input into landscape studies of type II compactifications (for recent results see [24] and references therein).

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